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SIMULATION-BASED PARAMETRIC RELIABILITY MODELING USING COVARIATE THEORY

Mr. Jon M. Wallace
Graduate Research Assistant, Aerospace Systems Design Laboratory, School of Aerospace Engineering, Georgia Institute of Technology

Dr. Dimitri N. Mavris
Associate Professor, Aerospace Systems Design Laboratory
School of Aerospace Engineering, Georgia Institute of Technology

ABSTRACT

Reliability analysis methods used for preliminary safety assessments of complex systems typically assume a predetermined and invariant set of input variable statistical properties. However, during the product development phase of the system and especially during in service operation, the characteristics of the random variables can themselves be subject to variation. Thus, the resulting failure probability distribution can vary greatly from early predictions. The objective of this paper is to explore a technique used to create a general parametric failure probability distribution as a function of key input variables. This technique is constructed around covariate theory which is the basis of the familiar Accelerated Life Testing and Proportional Hazards Modeling approaches. Where these approaches have traditionally been used with physical experiments, they are applied within this study to Monte Carlo simulation data generated using an available component modeling and simulation environment of a gas turbine airfoil limited by a single failure mode. Necessary modifications to the traditional form of the covariate approach are identified for application to controlled Monte Carlo simulation data. Implications to potential safety improvements early on in the product development phase are discussed.

KEYWORDS

Engine Reliability, Accelerated Life Testing, Failure Probability, Monte Carlo Simulation

INTRODUCTION

Reliability is an important customer metric for expensive and complex vehicle system designs. This is especially true for systems that require extended and manned flight operations. Yet, accurate system reliability assessment traditionally occurs far downstream of the conceptual design process when critical design decisions have already been made. Recent advances in multi-disciplinary analyses, design, optimization, probabilistic, and computational methods and capabilities now provide a medium that is much more conducive to considering reliability earlier in the design process of complex vehicles.

The objective of this paper is to demonstrate use of covariate theory to create a parametric representation of a component's probabilistic failure distribution as a function of several driving explanatory variables. Where response surface and other regression methods have been used to create a deterministic approximations of a given response, covariate theory can be applied to provide an approximation of the lifetime distribution not only as a function of the traditional index, time, but also as a function of additional variables that may affect the properties of the response distribution.

NOMENCLATURE

A_h	blade hub cross-sectional area
F_c	centrifugal force
$N_{fatigue}$	fatigue life
N	number of Monte Carlo simulations
r_t	blade tip radius
r_h	blade hub radius
T_c	internal blade cooling temperature
T_g	external turbine gas temperature
T_m	blade metal temperature
ΔT_a	ambient temperature change
ε	Monte Carlo simulation error
η_c	cooling effectiveness
η_f	fan efficiency
η_{hpc}	high pressure compressor efficiency
η_{cu}	combustor efficiency
η_{hpt}	high pressure turbine efficiency
η_{lpc}	low pressure compressor efficiency
η_{lpt}	low pressure turbine efficiency
ω	rotor speed
ρ	material density
σ_a	centrifugal stress amplitude
σ_c	centrifugal stress
σ_f', b	fatigue life fitting constants
σ_m	mean centrifugal stress

BACKGROUND

Parametric Distribution Determination

Before a covariate model of the suspect lifetime response can be created, the baseline distribution must either be specified or determined, for instance, through experimentation. Two methods are used with experimental lifetime data to statistically guide the process of selecting an appropriate distribution to represent the data: The Kolmogorov-Smirnov and Anderson-Darling EDF based methods[1,2,3,4,5]. EDF stands for Empirical Distribution Function which is a non-parametric determination of the cumulative failure, or survival, function and is the basis for many distribution test methods. However, unlike the traditional application of these distribution test methods where the data is generated through physical testing, it is generated within this study using computational simulation.

The Kolmogorov-Smirnov (K-S) goodness-of-fit test is useful in determining whether or not a given data set follows a hypothesized, continuous distribution. The K-S test is easy to implement and is completely general for any suspect parametric distribution when the empirical distribution is available. The K-S test statistic is defined as the maximum difference between the empirical cumulative distribution function, $\hat{F}(x)$ and the hypothesized fitted cumulative distribution function, $F_o(x)$. The null and alternative hypotheses for this test are:

$$H_o : F(x) = F_o(x) \quad H_1 : F(x) \neq F_o(x) \tag{1}$$

The test statistic for a complete uncensored data set is

$$D_n = \sup_x \left| \hat{F}(x) - F_o(x) \right| \tag{2}$$

where *sup* is an abbreviation for supremum. Thus, the test statistic is computed by looping over the entire data set. Larger values of D_n indicate a poorer fit of the hypothesized distribution. The test statistic formula is modified to a slightly different form for computational efficiency given as

$$D_n = \max\left\{ \max_{i=1,2,\ldots,n}\left(\frac{i}{n} - F_o\left(x_{(i)}\right) \right), \max_{i=1,2,\ldots,n}\left(F_o\left(x_{(i)}\right) - \frac{i-1}{n} \right) \right\} \tag{3}$$

where i is the i^{th} failure, $x_{(i)}$ is the i^{th} sequential lifetime or response value and n is the total number of realizations (samples) of the random variable. Algorithmically, one would fail to reject the null hypothesis and accept the hypothesized distribution with significant evidence if the computed K-S test statistic is less than the critical value of the statistics. The critical value is independent of the hypothesized distribution which is an appealing characteristic of the method. However, the K-S goodness of fit test is only valid for fully specified distributions. Therefore, if the same data for the goodness-of-fit test was used to estimate the parameters of the probability distribution, then the K-S statistic method cannot be used. Due primarily to this limitation, the Anderson-Darling test statistic is recommended.

The Anderson-Darling (A-D) test is a modification of the K-S test that weights the tails of the distribution resulting in a more sensitive test for goodness of fit of the hypothesized distribution. Contrary to the K-S statistic, the A-D test uses the hypothesized distribution to calculate the critical value of the test statistic. Therein lies the disadvantage. Although superior to the K-S statistic, the A-D method requires critical values to be computed unique to each distribution. Further, tables of critical values found in the literature have only been created for a few distributions such as the normal, lognormal, exponential, Weibull, extreme value type I, and logistic distributions. Fortunately, these distributions are some of the most widely used in statistics, especially in reliability modeling.

The A-D test statistic is defined as

$$A^2 = -n - S \tag{4}$$

where

$$S = \sum_{i=1}^{n} \frac{(2i-1)}{n}\left[\ln\left(F_o\left(x_{(i)}\right)\right) + \ln\left(1 - F_o\left(x_{(n+1-i)}\right)\right) \right] \tag{5}$$

In order to compare the A-D test statistic to its critical value, it must be modified by a factor which is dependent on the number of samples as well as the specific case encountered. For example, assume the case of testing the goodness-of-fit of a normal distribution for a given data set. If the location and scale (mean and variance) are to be estimated from the same data as that used for the test, then the appropriate modified A-D test statistic will be

$$A^2\left(1 + 4/n - 25/n^2\right) \tag{6}$$

The A-D method without using the critical value estimate has been applied in the literature to several distributions simultaneously. In this application, the distribution with the smallest A-D test statistic value would be the most appropriate distribution of the set to model the data. Once the most ideal distribution is selected, the critical value could then be calculated to statistically test for a specified significance level whether or not the distribution is truly representative of the data set. See the work by Stephens for a list of modifying factors as well as a complete account of widely excepted goodness-of-fit test techniques[1,2,3,4,5].

Covariate Models

Covariate models are used to represent the effect of suspect treatments in a probabilistic lifetime distribution or reliability model. A covariate is defined as a treatment or explanatory variable that influences the failure time of the component or item. A vector of covariates, $\boldsymbol{z}$, is chosen with each entry of the vector representing a unique explanatory variable. Typical covariates include those that represent mechanical forces, material properties, and environmental variables. There are two rather popular approaches for linking these covariates to the probability function. The first method, known as Accelerated Life Testing (ALT), is based on modifying the survivor function. The premise of the second approach, called Proportional Hazard Model (PHM), is to modify the hazard rate function to include the covariates.

In either method, the covariates are modeled by modifying the random variable with a 'link' function, $\psi(z)$, which is a function of the covariates. Usually, the link function chosen is of a log-linear form given as

$$\psi(z) = e^{\beta' z} \quad (7)$$

where β is an n by 1 vector of regression coefficients corresponding to each of the n covariates in the vector, $\mathbf{z}$. The appealing characteristic of this link function compared to others that have been proposed is that the exponential form is highly compatible with most traditional parametric distributions in that the model retains the necessary properties of a probability distribution. Also, the log-linear function is asymptotically stable across large ranges of the regression coefficients.

The resulting survivor function in the ALT approach becomes

$$S(t) = S_o\left(t\psi(z)\right) \quad (8)$$

where S_o is the baseline survivor distribution function determined at the nominal values of each of the potential covariates (i.e. z_i=0). This formulation imposes certain requirements on the link function. Namely, the function must be equal to unity at the nominal covariate setting, $\mathbf{z}$=0, and must be positive for any and all values of z. In essence, the ALT approach uses the link function to modify the time axis. The form of the log-linear link function used in both of the two approaches allows for the use of the familiar applied statistical model theory to determine the respective covariate regression coefficients, β_i. However, rather than use least squares regression, as is typical for statistical linear models and response surface equations, covariate models use maximum likelihood estimation (MLE) to estimate the covariate coefficients. The coefficients are treated as parameters of a multivariate distribution to be estimated using MLE. As a consequence, statistical significance tests of each coefficient in addition to whole model tests are readily computed resulting in a rapid importance ranking and error estimation of each coefficient.

MODELING AND SIMULATION ENVIRONMENT

Propulsion System Description

The propulsion system selected for this study is the CFM56 separate flow turbofan engine shown in Figure 1. The CFM56 is one of the most successful engine programs powering medium range transport aircraft such as the B737. The overall pressure and bypass ratios of the CFM56 are roughly 30:1 and 6:1, respectively. The maximum thrust at the sea-level static condition is around 20,000 lb_f.

The vehicle mission used in this study is typical of a B737 aircraft; although, it has been simplified to facilitate the demonstration of the proposed method. Only the cruise condition is considered for this study. The take-off and landing segments are modeled as discontinuous jumps in engine rotor speed between the shut-down and cruise segments of the operating profile. The cruise segment occurs at an altitude of 35,000 feet and a speed of Mach 0.745 as is typical for the B737-400. The mission parameters identified for this study are the cruise Altitude and Mach number.

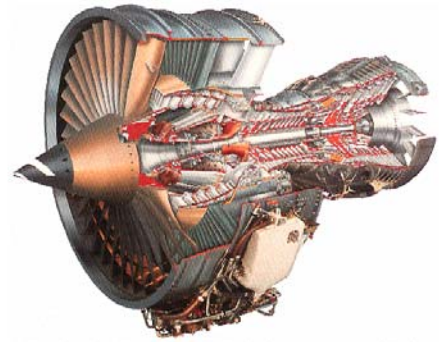

Figure 1: CFM56 Separate Flow Turbofan Engine Cutaway.

Multidisciplinary Turbine Blade Failure Analysis

The safe, reliable operation of the entire aircraft is highly dependent on the propulsion system utilized. One of the most critical components affecting the safety and reliability of turbine engines is the turbine blade. Failure of a turbine airfoil could cause a high-energy part to be released from the rotor system destroying the entire engine through a cascade of subsequent events. Such a situation could cause a catastrophic condition for the entire vehicle. Therefore, the propulsion system reliability is determined, within this study, by conducting a reliability assessment of a single turbine airfoil. There are numerous other failure conditions that can occur which would also affect system reliability and safety[6]; but, modeling and analyzing multiple critical-to-reliability components and failure mechanisms is not the focus of this study.

Turbine blade analysis requires the involvement of many disciplines and the use of advanced computational and experimental techniques which are beyond the scope of this study. Therefore, a first-order, integrated turbine blade multi-physics environment was constructed. The physical analysis structure, depicted in Figure 2, begins with system operational and ambient conditions providing input to a thermo-dynamic cycle model. This cycle model then is used to solve for the engine station state conditions as well as the mechanical speed of the rotors. A thermo-mechanical analysis is then conducted at the part level to determine the thermal and mechanical state of the part as required for the failure analysis.

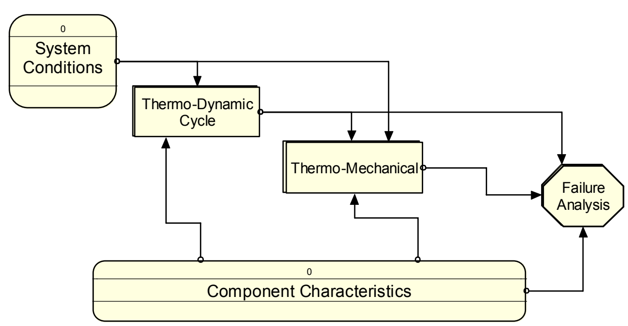

Figure 2: Turbine Blade Life Analysis Structure Matrix.

For the thermo-dynamic cycle analysis, a baseline NEPP thermodynamic cycle model of the CFM56 turbofan engine was utilized to compute the 1-D steady-state engine station and component properties[7,8]. The thermodynamic cycle parameters identified for this study are given in Table 1. Notice that three operational parameters representing the cruise flight condition are chosen along with the efficiency of each of the major cycle components. The required output provided by the cycle analysis is gross thrust, specific fuel consumption, high-pressure rotor speed, and turbine entrance and compressor exit temperatures. The gross thrust and specific fuel consumption responses were chosen as top-level metrics for comparison to the reliability prediction, which is also considered a top-level system metric. The remaining cycle responses are required as intermediate input for the subsequent thermo-mechanical and failure analyses.

After the core flow temperature and shaft speed are determined from the cycle analysis, a thermo-mechanical analysis is performed. The input for the thermo-mechanical analysis is given in Table 2. The thermal solid analysis required for determining the blade metal temperature is conducted using the definition of an empirical cooling effectiveness[9] given as

$$T_m = T_g - \eta_c \cdot (T_g - T_c) \tag{9}$$

This simple formula accounts for aerothermal and heat transfer behavior through this empirical cooling effectiveness parameter. A baseline expected cooling effectiveness of 0.63 is used, which is the value used to size the engine cycle model. The flow gas temperature, T_g, and coolant flow temperature, T_c, are simply the turbine entrance temperature and compressor exit temperature, respectively; which, are output from the cycle analysis. Radial variation of the temperature is not modeled in this analysis.

Although complex multi-axial states of stress exist in rotor airfoils, only the uni-axial bulk radial stress is considered here. The mechanical analysis for determining the bulk radial stress can be accomplished using Newton's second law applied to a rotating body. The centrifugal force produced anywhere in the airfoil can be determining by calculating the product of the mass of the supported material at the radius of interest and the square of the component rotational speed. Dividing this quantity by the airfoil cross-sectional area gives the bulk centrifugal stress at this radius. The maximum bulk centrifugal stress therefore occurs at the hub of the airfoil and is given as

$$\sigma_c = \frac{F_c}{A_h} = \rho\omega^2 (r_t - r_h) r_h \tag{10}$$

for a non-tapered geometry. The baseline material density and annulus area are provided as input. The rotor speed is calculated using the cycle model described earlier. With the thermo-mechanical conditions solved for, a failure analysis can then be conducted.

Table 1: Thermodynamic Cycle Input Values

Paramete	Categor	Unit	Baseline
Altitude	Operational	feet	3500
Mac	Operational	-	0.74
ΔT	Operational	°F	0
η_f	Component	-	0.88
η_{lpc}	Component	-	0.90
η_{hpc}	Component	-	0.87
η_{cu}	Component	-	0.99
η_{hpt}	Component	-	0.93
η_{lpt}	Component	-	0.93

Table 2: Thermo-Mechanical Input Values

Parameter	Category	Units	Baseline Value
η_c	Aerothermal	--	0.630
ρ	Material	lbm/in^3	0.300
r_T	Geometric	in	15.0
r_H	Geometric	in	13.0

Table 3: Failure Input Values

Parameter	Category	Units	Baseline Value
σ_f	Material	ksi	280
b	Material	--	-0.14

The failure condition, fatigue, can be highly involved but simplified here to facilitate the current study. Assuming that linear-elastic conditions exist, the straightforward stress-life, S-N, approach[10] can be utilized to estimate the number of cycles to failure given completely reversed, constant stress cycle amplitude. Morrow suggests a variation to the S-N approach to compensate for non-zero mean stress, which is given as[11]

$$N_{fatigue} = 0.5\left[\frac{\sigma_a}{\sigma_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)}\right]^{\frac{1}{b}} \tag{11}$$

where σ_a, σ_m, and σ_f are the stress amplitude, mean stress, and fatigue strength coefficient of the blade.

At this point, it is necessary to make another simplifying assumption with regards to the fatigue analysis. The Morrow low cycle fatigue function assumes that the cyclic stress amplitude is constant. Consequently, the stress cycle generated within the context of this problem is defined as the centrifugal stress cycling between the zero stress condition before starting the engine and the stress state at the cruise condition. Realistically, the stress-state is highly complex both temporally and spatially due to the varying operation conditions of the vehicle throughout its mission. Furthermore, material property and geometry non-linearity may also exist, which further complicate the analysis. Several experts within the area of mechanics of materials have focused considerable effort towards improving the prediction of the complex stress-strain state. For the purposes of demonstrating the proposed reliability method, a simple failure function is considered.

RESULTS

Identification of Baseline Distribution

The fatigue life data from 10,000 simulations of the multi-disciplinary turbine blade failure model was used to determine an appropriate parametric distribution to represent the distribution of the turbine blade fatigue life. The Anderson-Darling Test statistic is used to compare how well each distribution fits the given data. The smaller the A-D test statistic the better the fit of the parametric distribution. As shown in Figure 3, only the lognormal distribution properly represents the empirical data as it exhibited the smallest A-D statistic and visually captures the empirical data as shown by its probability plot. The other candidate distributions poorly model the data in the tail regions of the distributions. Thus, the popular Weibull and normal distributions are inferior to the lognormal distribution for this data set. Therefore, the baseline distribution chosen for this problem is the lognormal distribution. The parameter estimates were calculated using the maximum likelihood method and found to be 14.4912 and 1.6990 for the location and scale, respectively. The selection of the lognormal distribution can be validated using the A-D goodness-of-fit test provided earlier. Using equation (4) and (5) the adjusted A-D statistic is computed as 0.4571. The critical A-D value can be found using tables published by Stephens[1] for a normal distribution. Remember that the logarithm of the random variable, Fatigue life, follows a normal distribution. The critical value at a significance level of α=0.05 for a large number of simulations (i.e. n>>100) and for the case where the distribution parameters are *estimated* from the data is found to be 0.787. Therefore, with significant evidence the null hypothesis that the data follows a lognormal distribution with the computed parameter estimates is accepted and our selection of the appropriate distribution can be supported statistically. One can infer from this finding that the fatigue life, given normally distributed input, follows a lognormal distribution. This result can also be proven using analytical statistics.

The number of simulations used to generate the data set was originally chosen qualitatively as a balance between at least a moderate sample size, recommended for Monte Carlo simulations, and a sample size that can be processed efficiently to identify the appropriate life response distribution. However, it is well known that low probability predictions require a large sample size to achieve an acceptable accuracy when using the Monte Carlo simulation method. Shooman's formula[12] for estimating the error (%) of probability predictions from Monte Carlo data is useful for estimate the required number of Monte Carlo simulations. The formula is given as

$$\varepsilon = \sqrt{\frac{(1-P_f)}{N \cdot P_f}} x200 \tag{12}$$

where N is the number of simulations, and P_f is the probability level. Solving for N gives

$$N = \frac{1-P_f}{P_f}\left(\frac{\varepsilon}{200}\right)^{-2} \tag{13}$$

It is assumed that the lowest anticipated probability of failure level is 0.001 and an acceptable error is 5%. Thus, 1.5984E6 simulations are required to achieve this. Evaluating the location and scale parameter using the increased sample size had only a slight affect on the location and scale. They were recomputed as 14.5017 and 1.7047, respectively.

Should the required number of simulations be prohibitive to execute, there are several methods under the class of variance reduction techniques[13] that can be employed to improve the sampling points within the low probability regime. In this case, a mixed approach can be applied whereby a manageable number of Monte Carlo simulations can be executed then augmented with further variance reduction samples to improve the low probability prediction capability.

Predictions can now be made for the failure probability as a function of time. Using the lognormal parametric distribution, failure probabilities of as low as 1% and 0.1% can be expected at 37641 and 10236 cycles, respectively, with an error of at most 5% for the 0.1% probability level. Also, the predicted fatigue life at 0.1% can be converted to years by assuming that on average the aircraft takeoff and landing cycle occurs four times daily resulting in a 0.1% failure probability at 7.01 years of operation. Stated in another way, one can expect 1 in 1,000 turbine blades to have failed after 7 years. However, these predictions are valid only for the given mean values of such variables as Altitude and Mach number. If for example the mean cruise Altitude or Mach number were to be different, then this process would need to be repeated. The generation of a predictive model of the distribution function as affected by the values of suspect influential variables, known as covariates, is now demonstrated.

Covariate Model of Component Failure Distribution

The previous section determined that the baseline fatigue life distribution is best represented with a two-parameter lognormal distribution. As discussed earlier, one can use either the PHM or ALT methods to parameterize the failure distribution to so that it is a function of multiple explanatory (covariate) variables. This essentially is a parametric way to create a multivariate distribution function utilizing the baseline life distribution. The PHM method is advantageous because it can be used in a semi-parametric sense to prevent erroneously specifying an incorrect baseline distribution. However, our baseline analysis and subsequent A-D goodness-of-fit hypothesis test showed that the lognormal distribution function accurately modeled the fatigue life distribution. Therefore, the ALT method was selected.

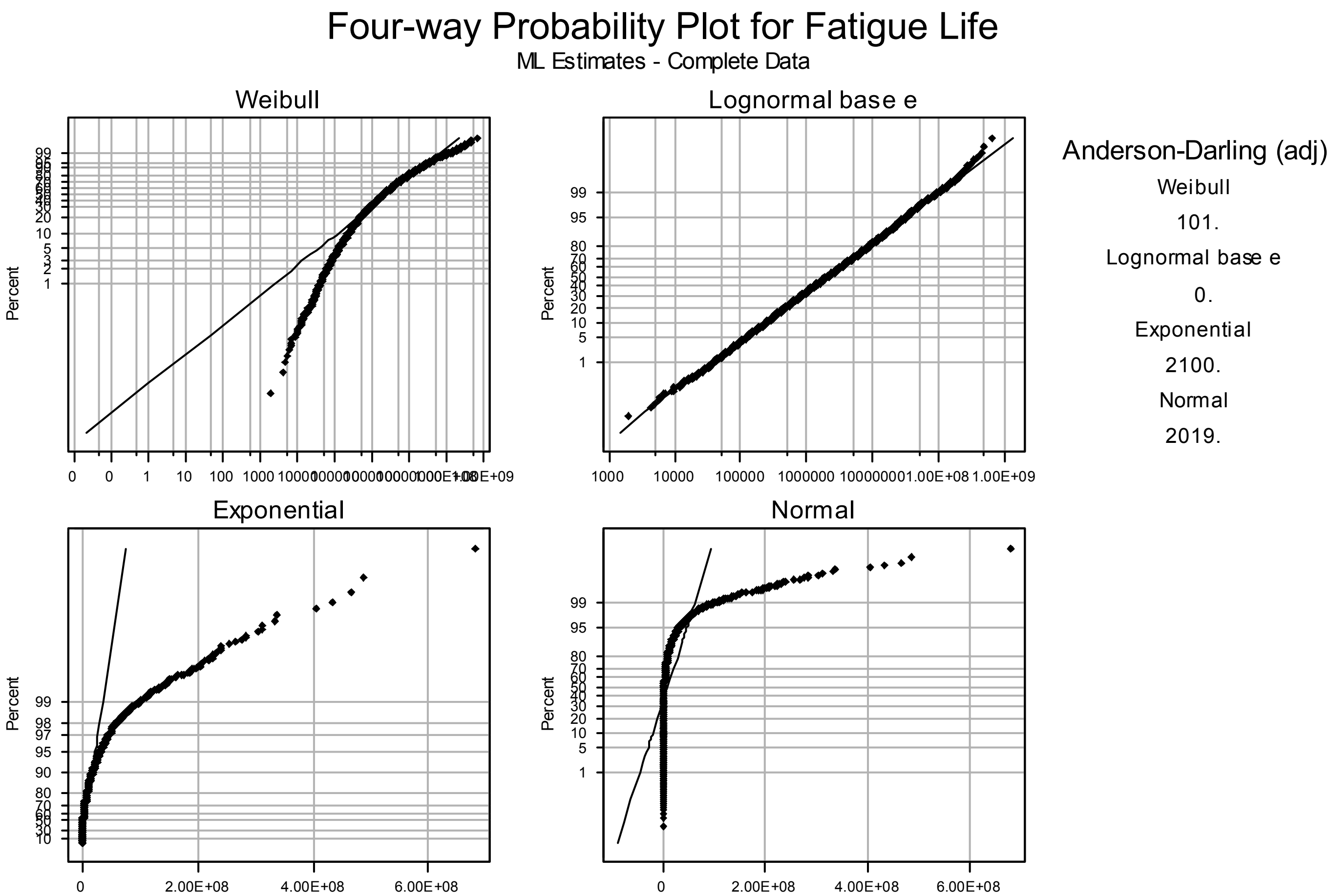

Figure 3: Probability plots of four hypothesized failure distributions

The ALT method requires the baseline survivor function which is determined in this case from the probability density function (PDF). The probability density function for the lognormal distribution is

$$f(x,\theta) = \frac{1}{\theta_2 \sqrt{2\pi}} e^{\frac{-(\ln x - \theta_1)^2}{2\theta_2}} \tag{14}$$

where θ_1 and θ_2 are the location and scale of the distribution, respectively, corresponding to the mean and variance of the natural logarithm of the fatigue life. Integrating over x gives the cumulative density function

$$F(x,\theta) = \int_0^x \frac{1}{\theta_2 \sqrt{2\pi}} e^{\frac{-(\ln \tau - \theta_1)^2}{2\theta_2}} d\tau \tag{15}$$

Since $\ln(x)$ is normally distributed, the cumulative density function (CDF) from equation (15) can be determined alternatively using

$$F(x,\theta) = P\left(Z \le \frac{\ln x - \theta_1}{\theta_2}\right) = \Phi\left(\frac{\ln x - \theta_1}{\theta_2}\right) \tag{16}$$

which is easily found using any standard normal table. The survivor function is the complement to the CDF given as

$$S(x,\theta) = 1 - F(x,\theta) \tag{17}$$

The log-quadratic link function is used to account for the covariate parameters in the ALT model. A quadratic, polynomial function is assumed for the exponent of the link function and is given as

$$g(\beta z) = \beta_1 z_1 + \beta_2 z_2 + \beta_3 z_1 z_2 + \beta_4 z_1^2 + \beta_5 z_2^2 \tag{19}$$

which upon substitution into the log-quadratic link function gives

$$\psi(z) = e^{g(\beta z)} = e^{\beta_1 z_1 + \beta_2 z_2 + \beta_3 z_1 z_2 + \beta_4 z_1^2 + \beta_5 z_2^2} \tag{20}$$

The PDF of the ALT model then is expressed as

$$f(x,\theta,\beta,z) = \frac{1}{\theta_2 \sqrt{2\pi}} e^{\frac{-(\ln(x \cdot \psi(z)) - \theta_1)^2}{2\theta_2}} \tag{22}$$

The ALT survivor function then becomes

$$S(x,\theta,\beta,z) = 1 - \Phi\left(\frac{\ln(x\psi(z)) - \theta_1}{\theta_2}\right) \tag{23}$$

The objective then is to acquire samples of the life at various values of the *covariate vector*, **z**, and use the maximum likelihood method to estimate the *coefficient vector* β, as well as the vector of baseline distribution parameters, θ.

Model Generation

As with any exercise of determining an appropriate model, a sample set which is indicative of the space to be modeled should be pursued. Determining the appropriate covariate model sample set, however, can be challenging. Meeker and Escobar recommend several considerations when designing the test matrix14. The considerations relevant to this problem are now discussed.

The coefficients of the ALT model are to be determined during the modeling process. Therefore, the accuracy of these coefficients is dependent on this process, especially the number of simulations at each vector value of the covariate variables. An initial estimate of the number of required simulations at each level of the covariates is helpful as a guide to designing the test matrix. Meeker and Eskobar recommend that the designer estimate the coefficient values from experience or available data. Then these temporary coefficient values should be utilized with the ALT model to allow the designer to rapidly explore both the levels of the covariate variables as well as the number of required simulations. Such an initial estimate can be used to evaluate different test schemes such as the spacing of covariate vector values as well as the number of simulations allocated to the various cases. Equal spacing and allocation of test points is the typical starting point. The authors of this paper also feel that under a limit of the number of experiments to run, one should allocate the number of experiments appropriately to each design of experiments case so that the variation of the initial maximum likelihood parameter estimates are minimized. For instance, if certain combinations of the covariates produce a lower probability level then one should allocate more experiments to these cases so that the accuracy at this probability level is improved. Also, one could alternatively use the classical power of test method to statistically determine the required number of total experiments[15]. For this study, a simple design of experiments test matrix with equally spaced covariate values was found to be sufficient.

With these considerations in mind, the design of experiments method was employed. To be able to model quadratic behavior, a 3-level full-factorial design was created using the two covariate variables, Altitude (z_1) and Mach number (z_2). The 3-level design was chosen here to be able to capture quadratic behavior should it exist. Also, the small number of candidate variables permitted a full-factorial design to be utilized. However, should a larger number of variables and levels be pursued a fractional factorial DOE, such as the popular Box-Benkin or Central Composite Design, is recommended.

The ranges of the two covariates are given in Table 4. Nine different DOE cases, as shown in Table 5, each with a unique value of Altitude and Mach number are required for the full-factorial design. However, the number of simulations to be executed for each combination of covariate variable values is somewhat subjective. Within this study the number of simulations for each case was chosen based on a fraction, say 1%, of the number suggested using Shooman's model (equation 13). This is an initial value which can be modified based on the adequacy of the model to be discussed later. An initial value of 10,000 simulations was chosen.

Table 4: Covariate Variable Ranges

Covariate	Minimum (-1)	Centerpoint (0)	Maximum (+1)
Altitude	34000	35000	36000
Mach	0.725	0.745	0.765

Table 5: Coded DOE Case Values

Case	Altitude	Mach
1	-1	-1
2	-1	0
3	-1	1
4	0	-1
5	0	0
6	0	1
7	1	-1
8	1	0
9	1	1

The results of the MLE fit of the covariate coefficients to 10,000 Monte Carlo simulations generated for each case are summarized in Table 6. The whole model test verifies that the covariates considered provide an exceptional model for representing the failure distribution while the standard error for each of the covariate coefficient estimates is minimal. However, the likelihood ratio tests for each coefficient show that the interaction and quadratic terms can be neglected in the final ALT model. Based on the model statistics produced by the maximum likelihood estimation approach, the number of case simulations was deemed to be sufficient.

Table 6: Parametric Survival (ALT) Fit Results

Parametric Survival Fit

Log-Normal Distribution

Whole Model Test

Model	-LogLikelihood	ChiSquare	DF	Prob>Chisq
Difference	7428.57462	14857.15	5	0.0000
Full	175722.538			
Reduced	183151.113			

Parameter Estimates

Term	Estimate	Std Error
Intercept	14.5049955	0.0127079
Altitude	0.814098	0.0069604
Mach	-0.3457485	0.0069604
Mach*Altitude	0.00711812	0.0085247
Altitude*Altitude	-0.0242726	0.0120558
Mach*Mach	0.01575175	0.0120558
Sigma	1.70494715	0.0040186

Effect Likelihood Ratio Tests

Source	Nparm	DF	L-R ChiSquare	Prob>ChiSq
Altitude	1	1	12734.8699	0.0000
Mach	1	1	2434.24084	0.0000
Mach*Altitude	1	1	0.69721637	0.4037
Altitude*Altitude	1	1	4.05351982	0.0441
Mach*Mach	1	1	1.70712408	0.1914

Validation of Covariate Model

As with any model, one should determine whether it is an adequate representation of the actual physical phenomena being modeled. However, with a probabilistic response this quickly becomes prohibitive for physical validation and more practical using computational simulation. Large simulation Monte Carlo analyses can be used to validate the model accuracy by verifying the probability or life prediction at various values of Altitude and Mach number. However, the choice of covariate value cases from which to evaluate the model or even how many cases to pursue is somewhat arbitrary. In the interest of computational efficiency the minimal number of cases necessary to assess the accuracy of the model is sought. For this study a random sample of validation cases was generated numbering roughly around half of the number of original cases used to create the model. The metric to be used to assess the accuracy of the model is the percent error of the probability predicted by the covariate model for three probability levels: 0.01, 0.1, and 0.50. Four cases were randomly generated to validate the model and are given in Table 7. The last three columns give the percent error of the model in predicting the life as compared to that which was computed using large sample Monte Carlo simulation. Notice that the maximum error across all of these points is 4.7% for the 1% probability of failure point, which is well within the 5% error originally specified for the baseline distribution probability prediction. Should this percent error be deemed to high, further simulation points could be run within this region of the probability space to improve the model. Therefore, the covariate model proved to be accurate across the probability space under consideration.

Table 7: Percent Error of Coded Validation Cases

			Probability Level (%)		
Case	Altitude	Mach	1	10	50
10	0.2076	0.4936	0.6	0.5	0.3
11	-0.4556	-0.1098	1.9	0.8	0.6
12	-0.6024	0.8636	4.0	1.4	1.7
13	-0.9695	-0.068	4.7	2.1	1.0

Parametric Life Prediction

Now that the ALT model has been created and validated, several exercises are possible. For instance, the life corresponding to a certain probability of failure, as shown by Figure 4, can be predicted for any combination of Altitude and Mach number within the specified ranges considered. Thus, an optimal setting of these two flight condition parameters could be searched that would maximize the life for a given acceptable probability of failure. Conversely, the probability of failure as a function of Altitude and Mach number for a given life, as shown by Figure 5, can be assessed. Further, the ALT model could serve as a robustness predictor enabling rapid identification of robust points from which to operate the aircraft. Additionally, the ALT model process could be repeated for design variables such as blade length and stress concentration (geometric concentration factor) thus creating a truly probabilistic design tool.

Another approach to create a parametric response function of reliability, a nondeterministic quantity, was pursued by the authors in an earlier study where the Response Surface Method (RSM) was utilized[16]. The response was reliability and the functional relationship was determined through a least squares solution of a quadratic formula relating flight and usage conditions and the reliability of a gas turbine component. Although the RSM produced an accurate response function of reliability, it was necessary to run at least 1,000,000 simulations per case to achieve such accuracy. With the covariate model explored within the current study, a comparable accuracy was achieved with much less simulations. Additionally, the RSM method does not provide statistical properties of the model as a function of the number of simulations executed. Thus, this potentially useful information is lost. With the covariate model approach, each simulation point is input into the maximum likelihood calculation and therefore used in determining the statistical properties of the assumed functional model. For example, the expected value and variation of each ALT coefficient is a direct function of the number of simulations executed as well as the assumed baseline probabilistic distribution. This is an important property of the covariate model approach which is necessary for creating a truly parametric life distribution function.

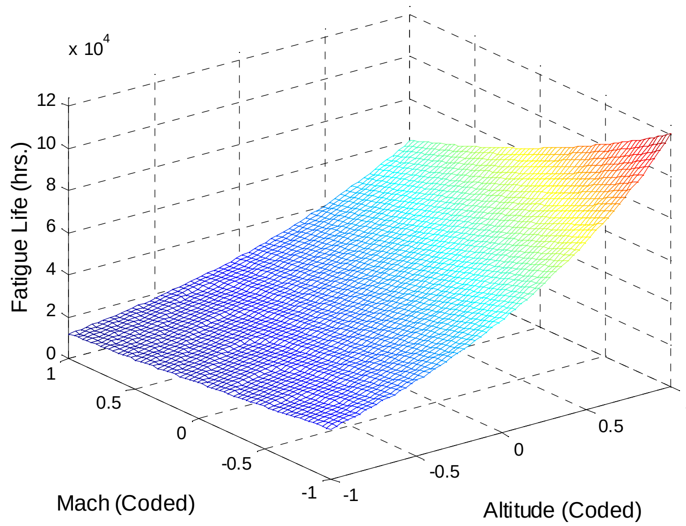

Figure 4: Life prediction using covariate model (P_f=1%).

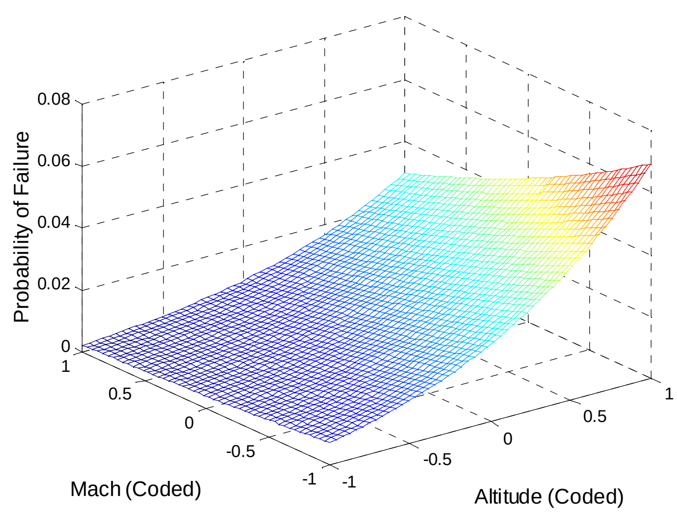

Figure 5: Probability prediction using covariate model (Life=50,000 hrs).

CONCLUSIONS

The covariate modeling method, popular in reliability testing, has been applied to simulation generated fatigue failure lives from an integrated gas turbine blade modeling and simulation environment. A multivariate probabilistic distribution model of the turbine fatigue life as a function of time, flight altitude and speed was produced using this method. Preliminary investigation of the fatigue failure behavior revealed lognormal behavior which was quantified using statistical hypothesis testing. General recommendations for implementing covariate models with simulation-based data inferred from the study include combining design of experiments and Shooman's law of Monte Carlo simulation error to assist in creating the fatigue life sampling test matrix. A random case generation procedure was executed to validate the accuracy of the model.

The model provides a functional relationship of the fatigue failure probabilistic distribution with respect to top-level system parameters and thus is a useful tool for conducting preliminary reliability design, optimization, and even prediction activities. Future work is anticipated to include developing a method of considering multiple failure modes simultaneously as well as a more refined and detailed procedure for utilizing the covariate model method with simulation results. A potential further advancement of this method could be formulation of the model generation steps to accept data generated from alternative probabilistic methods.

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